WOODY CALCULUS

Calculus 2 Formula, Method Selection, and Common Mistakes Guide

A printable reference for Calculus II and AP Calculus BC students who need to recognize the problem type, choose the correct method, set up the work cleanly, and execute accurately under exam pressure.

INTEGRATION	Decision rules for integration by parts, trig integrals, trig substitution, partial fractions, and improper integrals
APPLICATIONS	Area, washer and shell methods, work, hydrostatic force, arc length, and surface area
SERIES	Test-selection workflow, convergence conditions, power series, Taylor series, and error bounds
EXAM STRATEGY	Professor Woody's most common mistakes and a final exam-day checklist

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Recognize the problem. Choose the method. Execute accurately.

START HERE

How to Use This Guide

This guide is designed as a fast recognition and verification tool. It is not a substitute for full worked solutions. Use it to decide what family a problem belongs to, write the correct governing formula, and check the details that most often cause lost points.

1. Classify the problem.
   Identify whether the problem is an integration technique, an application of integration, a sequence or series, a power-series problem, or a parametric/polar problem.
2. Write the governing formula first.
   Do not begin algebra until the correct setup is visible on the page.
3. Execute one clean method.
   Avoid switching techniques halfway through unless the first choice truly fails.
4. Verify before stopping.
   Check bounds, signs, units, endpoint behavior, and whether the final result answers the exact question asked.

> **Professor Woody's Core Rule**
>
> A difficult Calculus 2 problem usually becomes manageable as soon as you correctly identify the problem type. Method selection comes before computation.

Guide Map

Section	Use It For
Integration Techniques	Decision guide, formulas, substitutions, decomposition, improper integrals
Applications of Integration	Area, volume, work, hydrostatic force, arc length, surface area
Parametric and Polar	Slopes, tangents, area, and arc length
Sequences and Series	Test selection, convergence criteria, power series, Taylor series
Common Mistakes	High-frequency errors that cost points
Exam Checklist	A final clean-work and verification routine

Full visual lessons and complete examples: brianwoody.com/math-library/

METHOD SELECTION

Integration Techniques Decision Guide

First question: can a simple u-substitution expose the derivative already present?

Before choosing a specialized method, look for an inside function together with a constant multiple of its derivative. If that structure is present, u-substitution is usually the cleanest first move.

Pattern You See	First Choice	Key Setup Check
Product with algebraic/log/inverse trig/exponential factors	Integration by parts	Choose u to simplify when differentiated; use tabular integration by parts for repeated polynomial differentiation.
Powers of sin x and cos x	Trig integrals	Odd power: save one factor. Both powers even: use half-angle identities.
Powers of tan x and sec x	Trig integrals	Even secant power: save $\sec^2 x$. Odd tangent power: save sec x tan x.
Radical with $a^2 - x^2$, $a^2 + x^2$, or $x^2 - a^2$	Trig substitution	Use $x = a \sin \theta$, $x = a \tan \theta$, or $x = a \sec \theta$ respectively.
Rational function P(x)/Q(x)	Partial fractions	Long divide first if degree P ≥ degree Q; then factor Q completely over the reals.
Infinite interval or vertical asymptote	Improper integral	Rewrite with limits; split at every interior singularity.
Repeated derivative pattern with exponentials or trig	Integration by parts	Cyclic behavior may require solving algebraically for the original integral.
Square root of a quadratic after completing the square	Complete the square	Then use a trig substitution or a standard inverse-trig form.
No clear pattern	Simplify first	Factor, divide, split fractions, use identities, or rewrite radicals before choosing a method.

Never force a method

A method is correct because the structure supports it - not because it appeared recently in class. Simplification often reveals the real technique.

CORE FORMULAS

Integration Techniques Reference

Integration by Parts

$$\int u\,dv = uv - \int v\,du$$

LIATE is a guideline, not a law: logarithmic, inverse trig, algebraic, trigonometric, exponential. Choose u so that differentiation makes the problem simpler.

Trig Integrals

Integral Pattern	Recognition Rule
$\int \sin^m x \cos^n x\,dx$	m odd: save sin x; n odd: save cos x; both even: half-angle identities
$\int \tan^m x \sec^n x\,dx$	n even: save $\sec^2 x$; m odd: save sec x tan x; otherwise rewrite with identities
Core identities	$\sin^2 x+\cos^2 x=1$, $1+\tan^2 x=\sec^2 x$, $1+\cot^2 x=\csc^2 x$
Half-angle	$\sin^2 x=(1-\cos 2x)/2$, $\cos^2 x=(1+\cos 2x)/2$

Trig Substitution

Radical Pattern	Substitution	Identity
$\sqrt{(a^2-x^2)}$	$x=a \sin \theta$	$1-\sin^2\theta=\cos^2\theta$
$\sqrt{(a^2+x^2)}$	$x=a \tan \theta$	$1+\tan^2\theta=\sec^2\theta$
$\sqrt{(x^2-a^2)}$	$x=a \sec \theta$	$\sec^2\theta-1=\tan^2\theta$

Triangle recovery

After integrating in θ, use a reference triangle or the defining substitution to return to x. State any domain assumptions needed for square roots and absolute values.

SETUP DETAILS

Rational Functions and Improper Integrals

Partial Fractions

Denominator Factor	Required Numerator Form
Distinct linear factor $(x-a)$	$A/(x-a)$
Repeated linear factor $(x-a)^k$	$A_1/(x-a)+A_2/(x-a)^2+\ldots+A_k/(x-a)^k$
Irreducible quadratic x^2+bx+c	$(Ax+B)/(x^2+bx+c)$
Repeated irreducible quadratic $q(x)^k$	$(A_1x+B_1)/q(x)+\ldots+(A_kx+B_k)/q(x)^k$

☐ Long divide before partial fractions when the rational function is improper.	☐ Factor the denominator completely over the real numbers.
☐ Include every repeated power in the decomposition.	☐ Use a linear numerator over each irreducible quadratic factor.

Improper Integrals

$$\int_a^{\infty} f(x)\,dx = \lim_{b\to\infty} \int_a^b f(x)\,dx$$

$$\int_a^c f(x)\,dx = \lim_{b\to c^-} \int_a^b f(x)\,dx$$

Interior singularity rule

If f has a vertical asymptote at c inside [a,b], split the integral at c. Both one-sided improper integrals must converge. If either one diverges, the original integral diverges.

$$\int_1^{\infty} 1/x^p\,dx \text{ converges exactly when } p>1$$

$$\int_0^{1} 1/x^p\,dx \text{ converges exactly when } p<1$$

FORMULA AND SETUP GUIDE

Applications of Integration

Problem Type	Formula	Setup Check
Area between curves	$A=\int(\text{top-bottom})\,dx$ or $A=\int(\text{right-left})\,dy$	Find intersections; split when the order changes.
Average value	$f_{avg} = (1/(b-a))\int_a^b f(x)\,dx$	Average height, not total area.
Disk / washer volume	$V=\pi\int(R^2-r^2)\,d(\text{variable})$	Slices perpendicular to the axis. Radii are distances.
Shell volume	$V=2\pi\int(\text{radius})(\text{height})\,d(\text{variable})$	Slices parallel to the axis. Height is top-bottom or right-left.
Work	$W=\int F(x)\,dx$	For springs, $F=kx$. For pumping, include weight density, volume, and lift distance.
Hydrostatic force	$F=\int \rho g(\text{depth})(\text{width})\,dy$	Depth is distance below the fluid surface; width comes from geometry.
Arc length $y=f(x)$	$L=\int_a^b \sqrt{1+[f'(x)]^2}\,dx$	Simplify inside the radical before integrating.
Surface area about x-axis	$S=2\pi\int_a^b \lvert y\rvert\sqrt{1+[y']^2}\,dx$	Radius is vertical distance to the axis.
Surface area about y-axis	$S=2\pi\int_a^b \lvert x\rvert\sqrt{1+[y']^2}\,dx$	Radius is horizontal distance to the axis.

Distance language prevents sign errors

Whenever a formula uses radius, depth, height, or lifting distance, write it as a geometric distance before simplifying. Distances must be nonnegative.

COMPARISON PAGE

Washer vs Shell and Arc Length vs Surface Area

Washer Method vs Shell Method

Feature	Washers	Shells
Slice direction	Perpendicular to axis	Parallel to axis
Geometric piece	Disk or washer	Cylindrical shell
Formula	$\pi(R^2-r^2)$	2π(radius)(height)
Best choice	When radii are easy and splitting is minimal	When height is easy and inversion/splitting is avoided
Common error	Using diameters instead of radii	Using width instead of shell radius

Arc Length vs Surface Area

Feature	Arc Length	Surface Area
What is accumulated?	Tiny distances ds	Tiny circular bands $2\pi r\,ds$
Core formula	$L=\int ds$	$S=2\pi\int r\,ds$
For $y=f(x)$	$ds=\sqrt{(1+[f']^2)}\,dx$	Multiply ds by the distance r to the axis
Units	Length units	Square units
Extra caution	Algebra inside the radical	Correct radius and axis of rotation

Caps are separate

The standard surface-area-of-revolution formula gives the curved lateral surface. Add flat endpoint caps only when the problem explicitly asks for the total area of a closed surface.

MOTION AND COORDINATE SYSTEMS

Parametric and Polar Formulas

Parametric Curves x=x(t), y=y(t)

$dy/dx = (dy/dt)/(dx/dt)$	**$d^2y/dx^2 = [d/dt(dy/dx)]/(dx/dt)$**
$L = \int \sqrt{(dx/dt)^2+(dy/dt)^2}\, dt$	**Area under curve = $\int y(t)x'(t)\, dt$**

□ Horizontal tangent: $dy/dt=0$ and $dx/dt\neq 0$.	□ Vertical tangent: $dx/dt=0$ and $dy/dt\neq 0$.
□ Convert parameter values into points by substituting into both $x(t)$ and $y(t)$.	□ Track direction by increasing t, not by the appearance of the curve alone.

Polar Curves r=r(θ)

$x=r \cos \theta$, $y=r \sin \theta$	**Area = $(1/2)\int r^2\, d\theta$**
$L = \int \sqrt{r^2+(dr/d\theta)^2}\, d\theta$	**$dy/dx = (r' \sin \theta+r \cos \theta)/(r' \cos \theta-r \sin \theta)$**

Polar bounds matter

Sketch or analyze one tracing interval before integrating. Symmetry can reduce work, but only after you know exactly how many times the curve is traced.

CONVERGENCE RECOGNITION

Infinite Series Test Decision Guide

Step zero: the Test for Divergence

Compute lim a_n. If the limit is not zero or does not exist, then $\sum a_n$ diverges. If the limit is zero, the test is inconclusive - continue.

1. Recognize a benchmark immediately.
   Geometric series and p-series should be identified before using a more complicated test.

2. Check signs.
   Positive terms invite comparison, limit comparison, integral, ratio, or root tests. Alternating signs require the alternating-series conditions and an absolute-convergence check.

3. Read dominant growth.
   Factorials and exponentials suggest the Ratio Test. Expressions raised to the nth power suggest the Root Test. Rational powers of n suggest comparison or limit comparison.

4. For power series, separate the jobs.
   Use Ratio or Root Test for the radius, then test each endpoint separately with ordinary series tests.

Pattern	Best First Test
ar^n or ar^{n-1}	Geometric series
$1/n^p$	p-series
Positive terms similar to a known benchmark	Comparison or Limit Comparison
Positive, continuous, decreasing f(n)	Integral Test
Factorials, exponentials, products	Ratio Test
Entire expression raised to n	Root Test
$(-1)^n b_n$ or $(-1)^{n+1} b_n$	Alternating Series Test, then absolute convergence
Power series $\sum c_n(x-a)^n$	Ratio/Root for radius; test endpoints separately

Do not use the most powerful test - use the most natural test

A clean geometric-series recognition is better than a long Ratio Test. The goal is correct, efficient classification.

CONDITIONS AND CONCLUSIONS

Series Tests Reference

Test	Standard Form	Conclusion
Geometric	$\sum ar^n$	Converges for $\lvert r\rvert<1$; sum=$a/(1-r)$. Diverges for $\lvert r\rvert\geq 1$.
p-Series	$\sum 1/n^p$	Converges for $p>1$; diverges for $p\leq 1$.
Direct Comparison	$0\leq a_n\leq b_n$	Smaller than a convergent series ⇒ convergent. Larger than a divergent series ⇒ divergent.
Limit Comparison	$\lim a_n/b_n = L$	If $0<L<\infty$, the two positive-term series have the same behavior.
Integral Test	$a_n=f(n)$	Requires positive, continuous, decreasing f; series and improper integral have same behavior.
Alternating Series	$(-1)^n b_n$	Converges if b_n decreases eventually and $b_n\to 0$.
Ratio Test	$L=\lim \lvert a_{n+1}/a_n\rvert$	$L<1$: absolute convergence. $L>1$ or ∞: divergence. $L=1$: inconclusive.
Root Test	$L=\lim \sqrt[n]{\lvert a_n\rvert}$	$L<1$: absolute convergence. $L>1$: divergence. $L=1$: inconclusive.

Absolute vs Conditional Convergence

If $\sum\lvert a_n\rvert$ converges, then $\sum a_n$ converges absolutely.

If $\sum a_n$ converges but $\sum\lvert a_n\rvert$ diverges, the convergence is conditional.

Comparison direction is not symmetric

Being smaller than a divergent series proves nothing. Being larger than a convergent series proves nothing. Match the inequality to the conclusion you need.

APPROXIMATION

Power Series, Taylor Series, and Radius of Convergence

Power Series Workflow

1. Identify the center a.
   Rewrite the series in powers of (x-a).
2. Find the radius R.
   Use the Ratio Test or Root Test and solve the resulting inequality in |x-a|.
3. Write the open interval.
   Convert |x-a|<R into a-R<x<a+R.
4. Test both endpoints separately.
   The Ratio and Root Tests are usually inconclusive at the endpoints.
5. State radius and interval.
   Give R and use interval notation with the correct endpoint inclusion.

Taylor series: $f(x)=\Sigma_{n=0}^{\infty} [f^{(n)}(a)/n!]\,(x-a)^n$

Maclaurin series: a=0

Essential Maclaurin Series

Function	Series	Interval
e^x	$\Sigma\, x^n/n!$	All real x
sin x	$\Sigma\, (-1)^n x^{2n+1}/(2n+1)!$	All real x
cos x	$\Sigma\, (-1)^n x^{2n}/(2n)!$	All real x
1/(1-x)	$\Sigma\, x^n$	$\|x\|<1$
ln(1+x)	$\Sigma\, (-1)^{n+1} x^n/n$, $n\geq 1$	$-1<x\leq 1$
arctan x	$\Sigma\, (-1)^n x^{2n+1}/(2n+1)$	$\|x\|\leq 1$ (conditional at endpoints)

Transform known series instead of differentiating forever

Substitute, multiply, integrate, or differentiate a known power series whenever possible. Always transform the interval of convergence and then recheck endpoints if needed.

KNOW WHICH TOOL APPLIES

Error Bounds and Approximation Accuracy

Alternating Series Error

$$|R_n| \le b_{n+1}$$

Use this when a series is alternating, the positive terms decrease, and the terms approach zero. The first omitted term controls the error.

Taylor / Lagrange Remainder

$$|R_n(x)| \le [M\,|x-a|^{n+1}]/(n+1)!$$

Here M is an upper bound for $|f^{(n+1)}(z)|$ on the entire interval between a and x.

Question	Alternating Series Error	Taylor Remainder
What is approximating?	An alternating infinite series by a partial sum	A function by a Taylor polynomial
What must be checked?	Alternation, decreasing b_n, $b_n \to 0$	A valid bound M for the next derivative
Error index	First omitted term b_{n+1}	Derivative order n+1 and factorial (n+1)!
Common mistake	Using the bound without checking the hypotheses	Using a point value instead of a maximum over the interval

Degree selection

When asked how many terms are needed, solve the error inequality for n. Do not stop after writing the bound - verify that your chosen n actually meets the requested tolerance.

POINT-SAVING CHECKLIST

Professor Woody's Most Common Calculus 2 Mistakes

☐ Integration: Choosing a technique before simplifying the integrand.	☐ Integration: Forgetting +C on an indefinite integral.
☐ u-Substitution: Changing the variable but not changing the bounds.	☐ Integration by Parts: Choosing u so the new integral is harder.
☐ Trig Integrals: Using the wrong identity for odd/even powers.	☐ Trig Substitution: Returning to x without resolving the correct sign or absolute value.
☐ Partial Fractions: Forgetting repeated powers or linear numerators over quadratics.	☐ Improper Integrals: Substituting ∞ or a singular endpoint directly instead of using limits.
☐ Area: Integrating top-bottom without checking where the curves switch order.	☐ Washer/Shell: Choosing the formula before choosing the slice direction.
☐ Washer Method: Using diameter where the formula requires radius.	☐ Shell Method: Confusing shell radius with shell height.
☐ Hydrostatic Force: Treating the coordinate y as the depth automatically.	☐ Arc Length: Expanding too early and missing a perfect-square simplification.
☐ Surface Area: Using x or y as the radius without measuring distance to the axis.	☐ Series: Using the Test for Divergence to claim convergence when $\lim a_n=0$.
☐ Comparison: Reversing the inequality direction needed for the conclusion.	☐ Alternating Series: Checking alternation but not decreasing terms and limit zero.
☐ Power Series: Finding the radius and forgetting to test both endpoints.	☐ Ratio/Root Test: Treating L=1 as a conclusion instead of inconclusive.
☐ Taylor Series: Forgetting the center (x-a), factorial, or derivative order.	☐ Error Bounds: Using the wrong next-term index or an invalid M.
☐ Parametric: Reporting t-values instead of the actual points on the curve.	☐ Polar: Integrating over bounds that trace the curve more than once.

The highest-value habit

Write one short setup line before every calculation: method, formula, bounds, and geometric quantities. That single line prevents many of the errors above.

FINAL VERIFICATION

Calculus 2 Exam-Day Workflow

1. Recognize the family.
   Name the problem type in the margin: integration technique, application, series test, power series, parametric, or polar.
2. Write the exact formula.
   Put the governing equation on the page before substituting quantities.
3. Define bounds and variables.
   State what the variable measures and why the bounds match the geometry or interval.
4. Simplify strategically.
   Look for factoring, identities, cancellation, perfect squares, or benchmark behavior before doing heavy algebra.
5. Execute cleanly.
   Keep equal signs aligned, avoid skipping algebra that changes signs or bounds, and preserve exact values.
6. Interpret the result.
   State convergence/divergence, units, interval notation, point coordinates, or the physical meaning requested.
7. Verify.
   Check sign, magnitude, endpoint behavior, units, and whether every condition of the theorem or test was satisfied.

Final 60-Second Scan

☐ Did I answer the exact question?	☐ Are all bounds and endpoints correct?
☐ Did I include +C where required?	☐ Are distances and radii nonnegative?
☐ Did I test every power-series endpoint?	☐ Did I state convergence type when relevant?
☐ Are units correct for length, area, volume, work, or force?	☐ Did I leave an exact answer unless a decimal was requested?

Exam pressure rule

When stuck, do not stare at the entire problem. Identify the next justified line. A correct setup often earns substantial credit even when the final computation is incomplete.

FULL LESSONS AND SUPPORT

Continue with Woody Calculus

This printable guide is the fast-reference layer. The Woody Calculus Math Library contains full visual explanations, complete worked examples, common-mistake sections, and professor checklists for the topics summarized here.

Resource	Direct Link
Calculus 2 Help	https://www.brianwoody.com/calculus-2-tutor/
Math Library	https://www.brianwoody.com/math-library/
Integration Techniques Help	https://www.brianwoody.com/integration-techniques-help/
Applications of Integration Help	https://www.brianwoody.com/applications-of-integration-help/
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About Brian M. Woody

Brian M. Woody is a Private Professor of Mathematics, former university lecturer, professional mathematician, and mathematical researcher with more than 25 years of university-level teaching experience.