Name: ______________________________

Section: ______________________________

Math 182, Practice Test 2. Spring 2020.

You may bring a piece of printer-sized paper ($8\frac{1}{2}$ by 11) with notes on both sides for the tests. Show all work for full credit, and cite any tests or theorems used, and the conditions therein. Use reduced fractions instead of decimals unless otherwise mentioned.

(1) Evaluate the limit as $n \to \infty$ of the sequence $\left\{\dfrac{2n+1}{n+2}\right\}_{n=1}^{\infty}$.

(2) Evaluate the limit as $n \to \infty$ of the sequence $\left\{\dfrac{2n+1}{n+2}\right\}_{n=1}^{\infty}$.

(3) Evaluate the limit as $n \to \infty$ of the sequence $\left\{(-1)^{n-1}\dfrac{n^2}{3n^2-1}\right\}_{n=1}^{\infty}$.

(4) Find a formula for the nth term of the following sequence.

$$\frac{5}{2}, -\frac{5}{4}, \frac{5}{6}, -\frac{5}{8}, \ldots$$

.

(5) Does the series converge or diverge? If it converges, what does it converge to?

$$\sum_{n=1}^{\infty} \frac{2^n}{3(5)^{n-1}}$$

(6) Does the series converge or diverge? If it converges, what does it converge to?

$$\sum_{n=1}^{\infty} \frac{9^{n-1}}{2^{2n}}$$

(7) Does the series converge or diverge? Cite any tests or theorems used.

$$\sum_{n=2}^{\infty} \frac{3n+2}{n-1}$$

(8) Does the series converge or diverge? Cite any tests or theorems used.

$$\sum_{n=1}^{\infty} \frac{2}{5\sqrt[3]{n}-1}$$

(9) Does the series converge or diverge? Cite any tests or theorems used.

$$\sum_{n=1}^{\infty} \frac{n}{e^{(n^2)}}$$

(10) Does the series converge or diverge? Cite any tests or theorems used.

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$$

(11) Does the series converge or diverge? Cite any tests or theorems used.

$$\sum_{n=1}^{\infty} \frac{2^n n^3}{n!}$$

(12) Find the radius and interval of convergence of the given power series.

$$\sum_{n=0}^{\infty} \frac{(x-1)^n}{\sqrt{n}}$$

(13) Find a power series representation for $f(x) = \dfrac{x^2}{1+x^3}$. Simplify.

Partial Solutions Note: here I give the final answer, but may not justify all tests/theorems.

1. 2

2. 0 (Note: I used the fact that IF $\lim_{n\to\infty} |a_n| = 0$, THEN $\lim_{n\to\infty} a_n = 0$. Notice that this is ONLY true for 0, which is why the limit in problem (3) fails to exist.)

3. Limit does not exist (i.e. the sequence diverges).

4. $a_n = (-1)^{n-1}\dfrac{5}{2n}$

5. Converges to 10. Geometric series.

6. Diverges ($r = \frac{9}{4} > 1$). Geometric series.

7. Diverges by the test for divergence (many other tests would also do).

8. Diverges. Either comparison test does well, as does the integral test.

9. Converges by the integral test. The function $f(x) = \dfrac{x}{e^{(x^2)}}$ is clearly positive and continuous for all $x > 0$, but checking that the function is decreasing requires computing $f'(x)$ which is negative for all $|x| > \frac{1}{4}$.

10. Converges by the alternating series test.

11. Radius of convergence 1. Interval of convergence $[0, 2)$.

12. $f(x) = \sum_{n=0}^{\infty}(-1)^n x^{3n+2}$