Fundamental Terms and Definitions

1. Determine the order of the following ordinary differential equations. Then state if the equations are linear or nonlinear using the following classification of a linear ordinary differential equation.

$a_n(x)y^{(n)} + a_{n-1}(x)y^{(n-1)} + \cdots + a_1(x)y' + a_0(x)y = g(x)$

(a) $(1-x)y''-7xy' + 5y = cos(x)$

(b) $t^8y^{(6)}-t^4y'' + 7y = 0$

(c) $\frac{d^2y}{dx^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^3}$

(d) $(sin(\theta))y'''-(cos(\theta))y' = 9$

2. State if the given differential equation is linear or nonlinear when:

$(y^8 - 1)\, dx + x\, dy = 0$

(a) The dependent variable is y.

(b) The dependent variable is x.

3. Check that the given functions are an explicit solution for the given differential equations.

(a) $9y' + y = 0,\ y = e^{-x/9}$

(b) $y''-4y' + 20y = 0,\ y = e^{2x}cos(4x)$

4. Verify that the given function $y = \phi(x)$ is an explicit solution of the given first-order differential equation. State the domain of $y = \phi(x)$ as a function, then state the interval of definition for the solution $y = \phi(x)$ to the given differential equation.

$(y-x)y' = y-x + 8,\ y = x + 4\sqrt{x + 5}$

5. Verify that the given function $y = \phi(x)$ is an explicit solution of the given first-order differential equation. State the domain of $y = \phi(x)$ as a function, then state the interval of definition for the solution $y = \phi(x)$ to the given differential equation.

$y' = 2xy^2$**,** $y = \frac{1}{(16-x^2)}$

6. Check that the given equation in an implicit solution to the following first-order differential equation. Then find an explicit solution for $X = \phi(t)$.

$\frac{dX}{dt} = (X-1)(1-2X)$**,** $t = ln\left(\frac{2X-1}{X-1}\right)$